文章编号：1000-4750(2005)Sup.-0240-17

连续—非连续结构应力分析现状与进展

*杨菊生，李九红，赵 钦
(西安理工大学，西安 710048)

摘 要：对工程结构应力分析中采用的连续介质模型、非连续介质模型、连续——非连续介质力学模型作了系统介绍。三种模型物理数值模拟的各种理论及数值方法，代表了当今结构应力分析的主流及发展方向。随着工程建设中高、大、精、新结构的出现，表明这个古老而又崭新的命题仍有无限广泛的发展前景。从七个方面论述了它的现状及发展特征：从过时观到现代新观念；从连续到非连续；从精确到数值近似；从有限元到无单元；从单一到统一；从标量无网格到复变量无网格；从独立到耦合，全面介绍了其内容和方法。

关键词：连续介质；数值模拟；应力分析；力学模型
中图分类号：TU311.3 **文献标识码：**A

STATE-OF-THE-ART OF STRESS ANALYSIS OF CONTINUOUS— DISCONTINUOUS STRUCTURES

*YANG Ju-sheng , LI Jiu-hong , ZHAO Qin
(Xi'an University of Technology, Xi'an 710048, China)

Abstract: Continuous, discontionuous, continuous—discontionuous medium mechanics models adopted in structural stress analysis are introduced systematically. Various theories and numerical methods in the three models which represent the mainstream and direction of stress analysis nowadays are discussed. With the advent of tall, large-scale, precise and new structures, research effort is still needed in the development of the three models. This paper describes the state of the art of stress analysis of continuous-discontinuous structures. It covers a great variety of issues including concepts, continuity to non-continuity, exact and approximate solutions and so forth.

Key words: continuous medium; numerical simulation; stress analysis; mechanics model

1 从过时到现代

工程结构应力分析科学领域内，当前存在着巨大的挑战，也面临着飞速发展的机遇。一方面随着现代工业技术的进步及人类挑战自然的努力，在航空、航天、航海、能源、建筑、水利、交通、环境等技术领域出现了大量高、大、精、新结构，如大型、高速飞机、轮船；火箭壳体；深埋地下工程；高速铁路及机车；三峡水电站；青藏铁路冻土工程；南水北调水电及岩石工程……等，原有的理论、方法和手段已不能满足该类新型结构物对强度、刚度、稳定性及经济性、可靠性提出的更高技术要求。另一方面，随着计算机技术的进步、推广及应用，又为传统意义上的结构应力分析研究开辟了广泛的发展空间及应用领域。过去在学术界存在的一种

作者简介：*杨菊生(1933)，男，陕西人，教授，主要从事固体力学及结构应力分析教学及研究工作(E-mail: yshubo@263.net);
李九红(1969)，男，山西人，副教授，博士，主要从事结构数值分析工作(E-mail: ljh@xaut.edu.cn)；
赵 钦(1978)，女，河北人，助教，硕士，主要从事结构数值分析工作.

结构应力分析相对于近代物理学成就而言毫无新意的过时观，已随着近代结构分析新理论体系的建立和计算机可视化技术、仿真模拟分析、多媒体网络技术的注入开创了生气勃勃的现代计算结构分析的新观念、新理论、新方法和新领域。

2 从连续到非连续

将研究对象用“连续介质”作为计算模型并用连续函数的解析方法研究其平衡及运动状态物理参数的数量关系(Ewler,1753)，构成了物理学中连续介质力学的理论基础[12,13]。由此而发展的液体力学、固体力学、气体力学等各个分支，皆以连续性假设为前提，对研究对象的物质组成(可统称为“材料”)及物理特性认识不断深化，各自形成了完整的理论体系，推动了工程技术不断进步。本文仅限于讨论可变形固体力学中的有关问题。

从一个螺丝钉到地球地壳，都是可变形的固体，工程科学中统称为“结构”。结构强度、刚度、稳定性是其研究内容的共性，按照对材料物理特性及结构性状的抽象，派生出了材料力学、结构力学、弹性力学、塑性力学、粘弹/塑性力学、流变体系力学、强度理论、流固耦合[1,2,3,10,11]……等力学分支。将它用于金属、混凝土、岩土、合金、橡胶等各种材料，可建立两种力学模型：

无缺陷模型：连续介质宏观模型。自十七世纪中叶(R.Hooke 1966)至第二次世界大战(R.V.Mises, 1913 及 T.Von.Karman,1939)，以弹性理论、塑性理论、稳定性理论为代表[17,18,19,20]；

有缺陷模型：非连续介质宏观模型。自二战至二十世纪 80 年代，以断裂力学(A.A.Griffith,1921)及损伤力学(L・M・Kachanov,1958)为代表[32,35,36,37,38,39,44]。

以结构工程使用最广泛的混凝土材料强度问题为例，混凝土在承载前已存在大量的微裂缝、加渣、气泡、孔穴等，严格说来，不能认为是连续的整体。工作过程中，屈服、裂缝出现及扩展、丧失承载力、变形等物理现象都可用来定义“混凝土破坏”。将其作为连续宏观模型处理，实验表明：在多轴荷载作用下，混凝土的强度不仅仅与单独压、拉、剪应力有关。其强度准则必须考虑各个应力分量之间的相互关系对混凝土强度的影响，这种破坏准则应表达为应力状态的函数。由于混凝土材料的实际特性和强度是一个非常复杂的问题，它与骨料和砂浆的物理力学性质、配合比、载荷特性等许多因素有关，在不同的载荷条件下所表现出的承载力也不同，故迄今为止还没有哪一种数学模型可以描述真实混凝土在所有条件下的强度问题。即使构造了这样一种数学模型，也会由于其过于复杂繁琐而难于应用于工程实际中去[61,62,63,241,242,247,248]。

2.1 强度准则

有关在多轴荷载作用下强度问题的研究[59]，最早始于伽得略于 17 世纪首先提出的最大主应力理论。但相当长时间混凝土的破坏理论发展较慢，只有在计算机推广应用后，在大量研究工作的基础上，混凝土的破坏理论才有了长足的发展[263~268]。

按照其理论体系及发展年代，各种强度准则可归纳为表 1 所示[234~236,253~255,257]。

西安交通大学俞宏茂教授于 1961 年提出了适用于等拉压强度材料的双剪应力屈服准则。历经三十多年的悉心研究，逐步发展为适用拉、压强度不同材料的双剪应力强度理论(1962，1983~1985)、双剪帽子模型(1986)，双剪多参数准则(1988~1990)，最后形成了具有统一力学模型、统一理论及统一的数学表达式，并可适用于多种材料的统一强度理论(1991)。上世纪 80 年代后期，又将其推广应用于结构的弹塑性分析，发展形成了双剪弹塑性本构模型和统一弹塑模型、双剪统一粘弹塑性本构模型。近年来，又有一些研究人员在此基础上作了进一步的工作，如用于断裂力学和损伤力学中去[258,259]。

双剪强度理论自成体系、内容丰富、描述统一、结构严谨，是我国学者对强度理论的重大贡献。它包含了国内外学者在过去的 20 多年所提出的一些主要强度理论，可用统一强度理论的形式概括描述。其理论及其应用价值不可低估，目前仍处于推广应用阶段。

2.2 本构关系

以连续介质宏观模型为基础的混凝土本构关系研究的理论体系包括：① 线弹性理论；② 非线性弹性理论；③ 弹塑性理论；④ 连续损伤力学理论；⑤ 断裂力学理论；⑥ 粘弹性理论；⑦ 内时理论[260,261,262,263,266,267,268]。

本构关系是描述材料软科学性质的一般表达式，如破坏准则、应力应变关系、高温特性、腐蚀特性等等。本文所涉及到的本构关系，主要是在二、三轴复杂荷载下反映混凝土材料受力特性的应力应关关系。它是在某些力学理论基础上建立所描述

介质的数学表达式。研究途径大致分为两大类：其一是侧重于理论研究，即以连续介质力学为基础，建立可被实验验证的能表达真实材料基本特性的理想化模型；其二是侧重实验研究，即以实验数据为根据，通过拟合的方式，给出反映材料综合特性的经验表达式。由于混凝土材料特性的复杂性，在其本构关系的研究中，实验研究是第一位的。而根据混凝土材料的物理参数从纯理论的推导得出一个完整的本构关系是极其繁难的[257,260,271,274]。

在线弹性理论基础上，通过描述材料随变形增长其弹性常数Ｅ，μ发生变化从而反映应力－应变非线性特性，即为非线弹性模型[,35,36,84,255,276]。由于该模型形式简单、便于应用，可直接根据实验结果进行，故广泛应用于工程结构分析[242,243,247,248]。比较有代表性的模型有：Darwin 和 Pecknold 模型、Kupfer 和模型、Ottonse 模型及在此基础上江见鲸等人提出的修正模型等等[234,236]。另外，还有违背能量守恒定律适用于各向异性、正交各向异性、横向各向异性材料的超弹性本构模型、三次弹性本构模型、次弹性本构模型[235]等等。

2.3 塑性条件 考虑材料塑性变形，必须确定以下基本原则，即：屈服条件、强化规律、流动法则、破坏准则[266,245,230,234]。

表 1 混凝土强度准则理论体系及发展

Table 1 Theory system and development in strength criteria of concrete

理论构架	强度理论和准则	创建人	年代	适用范围、条件及特点
古典强度理论	最大主应力理论(第一强度理论)	Galelio	17 世纪	脆性材料
		Rankine	1876 年	拉伸断裂
	最大应变理论(第二强度理论)	E.Mariotto	1682 年	单轴拉、压
	Mohr-Coulomb 强度理论 (第三强度理论)	C.A.Coulomb	1713 年	混凝土受弯破坏
		O.Mohr	1900 年	金属延性材料及混凝土脆性材料
		Mohr-Coulomb	C．20 初	混凝土、中等应力水平延性材料
		Druker-Prager	1952 年	克服了 M-C 准则破坏面不光滑缺点
	Tresca＆ Mises 屈服准则 (第四强度理论)	Tresca	1864 年	金属材料屈服，混凝土受压极限
		Von Mises	1913 年	材料进入塑性状态
三参数破坏准则	Bresle＆pister 三参数破坏准则	B.Bresle K.S.Pister	1958	不能反映 I_3 的影响，$\tau_{oct}\sim\sigma_{oct}$ 在偏平面上是圆
	Willam Warnke 三参数破坏准则	K.J.Willam E.P.Warnke	1975	$\tau_{oct}\sim\sigma_{oct}$ 在偏平面上是椭圆，可退化为圆
	Menetrey-Willam 准则	Menetrey	1995	可退化为 V.Mises.D-P,M-C
四参数破坏准则	Reimann 强度准则	Reimann	1965	子午线为抛物线
	Ottosen 准则	N.S.Ottosen	1975	包含全部偏应力张量不变量，实验吻合良好
	Hsich-Ting-chen 破坏准则	S.S.Hsich E.C.Ting W.F.Chen	1979	偏平面上为非圆形，可退化为 D~P.Mises 及最大拉应力准则
五参数破坏准则	Wilam-Warnke 五参数强度准则	K.J.Willam E.P.Warnke	1975	可描述低、高静水压力混凝土性能
	Kotsovos 强度准则	Kotsovos	1979	用指数型子午线和椭圆改进 W-W 准则的抛物线型子午线
	Podgorske 强度准则	Podgorski	1985	适用于混凝土、砂和粘土材料
	过一王五参数强度准则	过镇海 王传志	1991	适用于一切应力比和各种实验条件

(1) 屈服条件：混凝土材料没有明显的屈服极限，基于 Von Mises 屈服条件建立起来的相关联弹塑性模型称为 Prandtl-Reuss 材料；Drucker-Prager 考虑静水压力的影响对其作了改进，建立了由第一、第二应力不变量 I_1，J_2 表示的屈服条件；Lary 将 Mohr-Colomb 条件与最大拉应力准则联合使用，建立了 Lary 拉断 Mohr-Coulomb 材料的理想弹塑性模型。

(2) 强化规律。表示材料从屈服到破坏过程中变形发展的基本规律，是塑性理论的核心。许多模型都是通过正确地描述强化规律得到相应的本构关系。1975 年 W.F.Chen 最早建立的混凝土 Chen-Chen 塑性本构模型，把强化规律看作为破坏曲面的比例缩小，韩大健经改进提出了非均匀强化模型。在此基础上，Murray 等提出了三参数硬化模型(1979)；Hsieh-Chen 硬化模型；俞茂宏(1992)双剪弹塑性硬化本构模型，都进一步改进和完善了混凝土强化规律。

(3) 流动法则。反映了塑性应变增量与塑性的正交性。当塑性面取为强化面时为关联流动法则，否则便是非关联流动法则。文献[269]通过分析认为关联流动法则不能正确预报混凝土塑性体积变化规律，因而发展了非关联流动的本构模型。这一方面模型有：Vermeer-Borst(1984)非关联流动的本构模型、Vermeer-Bost(1984)非相关流理想弹塑性本构模型、Pietruszczak-Jiang-Mirza(1988)混合硬化模型、Han-Chen 混合硬化模型等。

(4) 内时理论。Valauis 于 1971 年提出的内时理论模型，不以屈服面、硬化规律、流动法则为基本条件，认为塑性、粘塑性等类耗散材料内任一点现时应力状态是该点邻域内整个变形和温度历史的泛函，该历史用取决于变形中的材料特性和变形程度的内蕴时间标准 Z 来度量。其理论基础是以内变量理论为基本网络的不可逆热力学，形式上代表着一种特殊形式的粘塑性理论。Bazant 等曾将其用到混凝土本构关系中，概括了应变软化、非弹性体积膨胀、静止压力敏感性等基本特性，但难以推广应用。

2.4 非连续损伤－断裂宏观模型

混凝土在自然状态下就是一种有缺陷的多孔隙介质，在受荷载后，其破坏是由微观裂缝的起裂一发展一扩展直至出现宏观裂缝的脆性断裂过程[38,39,40,41,45,47,61,62,63,68,69,70,71]。应用上述连续介质力学理论建立的本构模型，有广泛的使用价值，但严格的说，都不能反映混凝土材料的破坏本质。建立在非连续宏观模型基础上的损伤力学及断裂力学，在内变量耗散理论及热力学基本定律基础上，用微观、宏观、宏微观结合的方法的变革[79,80,83]。近三十年来，随着实验手段的改进及仿真模拟的发展，又取得了长足进步，各种理论方法著作，揭示有缺陷损伤－破坏的内在规律，从而为破坏准则及本构关系的研究开创了新的局面，带来了认识其破坏机理、建立物理及数学模型、完善数值论文及研究成果不胜枚数[40,45,48,49,50,56,57,58]，但能全面反映混凝土这种复合材料特征的成果，尚不成熟，本文不再引申和讨论。

综上所述，目前还没有哪种理论被公认是完全符合混凝土材料特征的准确描述，但各种理论又各具特色，分别适用于一定条件及范围，并取得了一些有工程价值的成果。在理论发展的过程中，又出现了许多组合理论，如弹塑性损伤断裂模型；塑型断裂模型；塑型损伤模型，以及边界面模型、微平面模型、神经网络材料模型等。其结果是理想模型日趋复杂，而对提高精度和在工程中推广应用方面贡献不大。

3 从精确到近似

工程结构的应力分析，不管把实际问题理想化为什么样的物理模型，在数学上，都归结为在给定边界条件下的工程科学问题控制微分方程，即微分方程的边值问题。统一数学描述为：$L\phi = q$ (在 D 域内)；$\phi_s = \widetilde{\phi}$ (在 S 边界上，$S \in D$)；q 为已知或未知的 ϕ 的函数，L 为线性或非线性微分算子。

边值问题的解答，可有三种途径：解析解、数值解及实验解。解析解即边值问题的完全精确解，对于弹性、塑性、粘性静动力学问题仅可对简单边界典型问题得到数学上的精确解答[1,2,3,4]；在计算机面世以前，对复杂边界实际问题，只能借助于机械、光学、电磁学等物理手段模拟其类比实验解答，如电比拟法、光测法、薄膜比拟法等[4,10,11]。连续介质问题的数值解答分为微分方程数值解及离散模型数值解两大类。微分方程数值解是将微分方程在数学上进到“离散”，将其方程转化为线性或非线性代数方程，从而把无限自由度(连续介质)用有限自由度(离散模型)表达，求出满足边界条件的离散模型。在给定点(或网格)上近似值，如数值积分法、

加权余量法、有限差分法、瑞利－李兹法、变分法、伽辽金法等；或将微分方程降价为边界积分方程，用直接积分或间接积分的方法求边界积分方程的数值解。离散模型数值解，则是对计算模型(求解域)进行物理－几何上的分割，将连续体用有限元单元(域内或边界上)在有限个结点上联结为离散集合体，从而用代数方程求出结点或单元内的数值近似解[5,6,7,8,9]。这种从精确解到近似解的求解过程，促进了工程技术的发展，是人类认识客观世界本质的一个飞跃。从客观意义上讲，它是定量评价未来客观世界一切过程的基本方法之一，具有不可估量的技术经济和社会发展的意义。

4 从单一到统一

不连续介质离散模型数值近似解最早出现于岩体工程中分片刚体模型，如刚体元法(Cundall,1971)及块体元法(石根华，1983)，不考虑单元变形，皆属于刚体离散单元模型。其实质都基于分片刚体模型，可反映岩体中的断层、夹层、裂隙、节理不连续结构面上的位移及运动性态[21,22,24,25]。计入单元变形的变形体离散单元模型，如刚体弹簧元法(Kawai,1977)及变形块体元法(石根华，1992)皆是分片变形体模型[26,27,28]。刚体弹簧元法是将块体元变形用累积在弹簧上的变形表示，其支配方程类似于有限元，故又称为刚性有限元法。变形块体元法采用不连续变形块体模型兼有有限元的特点，块体运动分析类似刚体离散单元，而每一时段内的求解类似于有限元。在上述思路基础上，卓家寿基于 Kawai 刚体弹簧元思路提出了界面元离散模型(1993)，以界面元替代弹簧反映不同类型单元的不连续变形并导出了各向异性体单元交界面上任一点的界面应力公式，可用来解决各向同/异性体连续－非连续介质工程结构的数值分析问题[28,29,30,85,86,87]。而流形元法(石根华，1991；林德璋，1992)、无单元法或称无网格法(Belytschko,1994)，统一数值流形元法(石根华，1996)，则为求解统一连续－非连续问题开辟另一新的途径[91,96,98,135]。

5 从有限元到无单元

自 1960 年 R.W.Clough 奠定了有限元法基本框架后，经过 O.C.Zienkiewicz (1965)、E.L.Wilson (1973)等人在一般场问题及软件开发方面的卓越工作[5,6,7,8,9]，至 1975 年，K.H.Huebner 在其“The Finife Element for Engineers”一书中说：“从为一种分析技术来讲，有限元法已达到不可能期望再有别的戏剧性的发展或突破了。当然未来的发展将包括在许多实际问题中更广泛的应用，增加对特殊重要方面的理解，进而提高和完善有限元法的基本技巧”[5]。近三十多年来，J.H.Argyris、C.A.Brebbia、卡卞学璜等学者在数值物理模型、数学理论分析、各个领域中的推广应用及软件市场开发等方面的卓越贡献皆验证了 Huebner 的这一论断[14,15,16]。

在细、微观上，所有变形固体都是非连续介质，宏观连续介质的非连续性，包含两类基本问题：其一是含有节理、裂缝构造的非连续体，破坏表现为不连续面张开及剪切滑移；其二是连续体中裂缝的产生和扩展。

随着耗散、分叉、分形、浑沌、模糊等理论的形成及发展，在连续介质有限元基本思想基础上，如上所述，自上世纪 90 年代初期，发展了非连续介质固体力学各种数值算法，进而发展了统一连续和非连续介质问题的数值方法[30,27,91]。解决宏观非连续性问题，当前主要有五种技术手段，即：弹塑性断裂力学模型数值方法(FEM、BEM、WRM、COV 等)；弹塑性损伤力学模型数值方法(PDM、BEM、WRM、COV 等)；不连续块体模型数值方法(DEM、SEM、RSM、DDM、IEM)等；数值流形元法(FEM＋COVER)；无单元或无网络法(NEM)。其中，数值流形元法是有限元的延伸、扩大发展[96,97]。它引入了数学及物理两种网格，来定义数值解精度(如用三角形单元)及求解域边界及裂缝面；用数学及物理两种覆盖(Cover)来代替“单元”及“结点”；以三个物理覆盖之交集定义了“流形单元”，即用物理覆盖代替单元的节点，用覆盖的交线代替单元的边界，用覆盖的交集代替单元。被节理线、裂缝面、边界等隔离开的流形单元的公共线间可有不同节点，故可反映开裂，表现出非连续性[279,280,281,282,203]。从而实现了连续性与非连续性的统一。而无单元法，则完全没有“单元”及网格的概念。

6 从无网格法到复变量无网格法

无网格法产生至今已逾二十余年，由于无网格法在场变量的逼近、边界条件的引入、能量泛出积分等方面存在的灵活性，相继出现了不同命名的多种无网格方法。在其发展过程中，多数学者所提出

的数学格式都是以移动最小二乘法形成无网格方法的逼近函数。表 2 对已见诸报导的各种方法及特点作了综合比较[103~130,133~140]。

目前较流行的是无网格 Galerkim 法(即 EFGM)，它以移动最小二乘法构造插值函数[196,197]，从微分方程的弱变分原理出发，导出求解问题的代数方程。其优点是自适应分析中不需重构网格；前处理简单；计算结果光滑连续；数值精度较高。作为一种新兴的数值计算方法，理论上具有开创性并初步形成了完整的构架及体系。但还存在着以下问题[187,188,278]：

(1) 作为无需背景网格的点积分法还缺乏必要的理论支持，精度有所降低；

(2) 运算中每一点都需计算一次形函数及其导数，每次都涉及矩阵求逆及连乘，在每个背景积分网格中皆需使用高阶高斯积分，计算量大、费机时；

(3) 影响求解精度的因素多，包括节点分布密度、基底函数的阶次、权函数的选取及其影响域大小；边界条件引入的不确定等；

针对目前使用标量的移动最小二乘法形成的无网格方法配点过多的问题，近期提出了对向量函数逼近的复变量移动最小二乘法[131,277]，其优点是所取试函数中所含待定常数减少，形成的二维问题的无网格方法可取较少的节点，对任一场点而言，其紧支域中所含的最小节点数就大大减少了；同时不形成病态方程组、精度高，所形成的无网格方法计算量小。

采用基于正交基函数的复变量移动最小二乘法建立的复变量无网格法，将具有配点少、精度高、计算速度快的优点，可有效地解决使用变量的无网格法配点过多。计算速度慢、容易形成病态方程组的缺点。为了较好地模拟裂纹尖端场的局部化现象，在复变量无网格法基础上，利用裂纹尖端位移场的解析解扩展复变量无网格法的基函数，提出了扩展的复变量无网格方法，可较好地解决含裂纹的问题，提高了复变量无网格法求解裂纹问题的精度[277]。

7 从独立到耦合

无网格法具有有限元法的一些优势，但它的缺点正好是有限元法的优点，采用二者耦合的方法，“取长补短、优势互补”，将为其应用开拓广阔前景。

有限元法从理论——方法——应用——软件开发诸方面已相当完善，随着 CAD 及可视化、多媒体、网络技术的发展及在计算分析中的应用，有限元法可视化分析软件系统在数量上、应用价值上皆居主流和领导地位。国际知名的有限元软件约有 400 多种，其中较著名的有十几个，如 NASTRAN、MARC、ASKA、ADINA、GTSTRUDL、ANSYS、ABAQUS、SAP……等，其强大的计算、设计内核及可视化程度相当高的前、后处理系统，已被国际工程项目开发认可并作为标准[283]。充分利用这一市场优势，不断维护、完善、提高、扩大有限元软件的功能及使用范围，将具有重大的技术经济价值。

把有限元法与加权残余法耦合或与边界元法耦合，已取得了可喜成果[209,225,284,285,286,287,288,290]。Belytschko 最早提出了无网格法与有限元法的耦合方法，将求解域划分为无网格子域及有限元法子域，在两子域的界面元定义适当的耦合位移函数保证两子域界面上的位移连续；Huerta 则采用直接耦合的方式，在交界域内将相应的无网格法和有限元法的节点各自形成的逼近或插值函数迭加形成交界区域内任意点的逼近函数。程玉民、李九红则是将复变量无网格方法与有限元法耦合，将求解域划分为复变量无网格子域及有限元子域，各子域分别用复变量无网格法及有限元法建立基本方程，在子域交界处设置虚拟有限元，通过两子域交界面条件形成统一的求解方程。该法保留了无网格法可随意配点的优点；交界面可不规则；提高了无网格子域内的求解精度，从而提高了问题的整体求解精度，基本解决了 Belytschko 及 Huerta 耦合法存在的问题；但在交界面附近区域误差较大[277]。

8 结论

(1) 连续介质数值方法，以有限元法为代表，在理论、应用、软件开发等方面已臻成熟。重要的问题是扩大其应用领域，继续完善软件功能，特别是加强可视化及仿真技术的应用，使其界面更加友好。

(2) 非连续介质数值方法，在理论上尚不够成熟，以损伤力学及断裂力学为代表的处理有缺陷模型的理论及方法，仍需在大量实验基础上继续完善，构成统一的破坏数值模型及理论框架；

表 2 无网格方法小结

Table 2 Brief summary of meshless methods

时 间	研究学者代表	名 称	近似函数	离散方案
1975 年	Perrone,Kao	广义有限差分法 (General Finite Difference Method for Arbitray Meshes)	核函数	配点法
1977 年	Lucyt	光滑质点流体动力学方法(SPH) (Smoothed Particle Hydrodynamics)		
1992 年	Nayroles	散射元法(DEM) (Diffuse Element Method)	移动最小二乘法	Galerkin 法
1994 年	Belytschko	无网格 Galerkin 法(EFGM) (The Element-Free Galerkin Method)	移动最小二乘法	Galerkin 法
1994 年	Amaratunga K	小波 Galerkin 法(WGM) (Wavelet-Galerkin Method)	小波级数	Galerkin 法
1995 年	Liu W K	再生核质点法(RKPM) (Reproducing Kernel Particle Method)	再生核近似	Galerkin 法
1995 年	Liu	多尺度再生核质点法(MSRKPM) (Multi Scale Reproducing Kernel Particle Method)	再生核近似	Galerkin 法
1995 年	Liu W K	小波质点法(WPM) (Wavelet Particle method)		
1995 年	Oden	Hp 云团(Clouds)法(HPCM) (Hp Clouds Method)	单位分解法	配点法
1996 年	Melenk J M	单位分解法(PUM) (Partition of Unity Method)	单位分解法	Galerkin 法
1996 年	Melenk J M	单位分解有限元法(PUFEM) (Partition of Unity Finite Element Method)	单位分解有限元法	
1996 年	Onate	有限点法(FPM) (The Finite Point Method)	移动最小二乘法	配点法
1996 年	Liu W K	移动最小二乘积分核法(MLSRKM) (Moving Least-Square Reproducing Kernel Method)	移动最小二乘积分核法	
1996 年	Liszka	Hp 无网格云团法(HPMCM) (Hp Meshless Clouds Method)	移动最小二乘法	配点法
1997 年	石根华	无单元流行法(MM) (Manifold Method)		
1998 年	Sukumar N	自然单元法(NEM) (Natural Element Method)		
1998 年	Oden	新型 Hp 有限元法(NCBHPFEM) (New Clo uds-Based Hp FEM)	有限元形函数	
1998 年	Arluri S N	无网格局部伽辽金法(MLPGM) (Meshless Local Petro-Galerkin Method)	移动最小二乘法	Petro-Galerkin 法
1998 年	Zhu T	局部边界积分法(MLBIEM) (Meshless Local Boundary Integral Equation Method)	移动最小二乘法	Petro-Galerkin 法
2000 年	Babuska	广义有限元法(GFEM) (Generalized Finite Element Method, GFEM)		
2000 年	田荣	有限覆盖无单元法(FCEFM) (Finite-Cover Eleme nt-Free Method)		
2000 年	张雄	紧支径向基函数无网格法 (Meshless Method Based on Collocation with Radial Basis Function)	紧支径向基函数	配点法
2001 年	Ohs R R	无网格配点法(PCM) (Meshless Point Collocation Method)	再生核近似	配点法
2001 年	张见明	杂交边界点法(HBPM) (Hybrid Boundry Point Method)	移动最小二乘法	Galerkin 法
2001 年	张雄	最小二乘配点无网格法(LSCMM) (Least-Square Collocation Meshless Method)	移动最小二乘法	最小二乘配点法
2001 年	张雄	加权最小二乘无网格法(WLSMM) (Weight Least-Square Meshless Method)	移动最小二乘法	加权最小二乘法
2004 年	程玉民, 李九红	复变量无网格方法 (Meshless Method with Complex Variables)	复变量移动最小二乘法	Galerkin 法

(3) 流形元法及无网格方法，为建立连续——非连续统一数学模型奠定了基础，但仅有二维问题的初步成果，没有可供推广的成熟的软件，实际工程的应用还处在探索阶段，“任重道远”，要达到有限元、边界元等数值方法的应用水平，还相差甚远。

(4) 如何在丰富众多高效的 FEM 商业软件中，引入处理非连续问题的功能，作到连续与非连续的统一，是当前可变形固体力学应着重考虑的问题。从这一角度出发，数值流形元法、无单元法与 FEM 耦合问题的研究及相应的软件开发，更具有开拓发展的价值。

参考文献：

[1] 钱伟长，叶开元．弹性力学[M]．北京：科学出版社，1956.
Qian Weichang, Ye Kaiyuan. Elastic mechanics [M]. Beijing: Science Press, 1956. (in Chinese)

[2] H. И. 穆斯海里什维利，赵惠元译．数学弹性力学的几个基本问题[M]．北京：科学出版社，1958.
Н·И·Мусхелищвилй. Some basic problems of mathematic elastic mechanics [M]. Beijing: Science Press, 1958. (in Russian)

[3] 徐芝纶．弹性力学[M]．北京：高教出版社，1990.
Xu Zhilun. Elastic mechanics [M]. Beijing: Higher Education Press, 1990. (in Chinese)

[4] L. E. Malvern. Introduction to the mechanics of a continuous medium [M]. Prentice-Hall, Inc., 1969.

[5] 埃伯纳．工程师用有限元素法[M]．北京：国防工业出版社，1983.
K. H. Huebner. The FEM for Engineers [M]. Beijing: National Defence Press, 1983. (in Chinese)

[6] Ivar Holand and Kolben Bell. FEM in stress analysis [M]. Prentice Hall, 1972.

[7] C. A. Brebbia, Finite Element Systems (A Handbook) [M]. McGraw-Hill, 1985.

[8] S. S. Rao. The FEM in Engineering [M]. McGraw-Hill, 1982.

[9] O.C.Zienkiewiz. The finite element method in engineering science [M]. McGraw-Hill, 1971.

[10] 铁木辛柯，古地尔．弹性理论[M]．北京：教育出版社，1951.
S.Timoshenko, J.N.Goodier. Theory of Elasticity [M]. Beijing: Edu. Press, 1951. (in Chinese)

[11] O.C.Zienkiewicz and Y.K.Cheung. The FEM in structural and continuum mechanics [M]. McGraw-Hill, 1967.

[12] 冯康，石钟慈．弹性结构的数学理论[M]．北京：科学出版社，1984.
Feng Kang, Shi Zhongci. The mathematic theory of elasticity structures [M]. Beijing: Science Press, 1984. (in Chinese)

[13] 钱伟长．变分法及有限元[M]．北京：科学出版社，1980.
Qian Weichang.Variational principles and FEM [M]. Beijing: Science Press, 1980. (in Chinese)

[14] 徐次达，华伯浩．固体力学有限元理论，方法和程序[M]．北京：水利水电出版社，1983.
Xu Cida, Hua Bohao. The theory, method and program of FEM in solid mechanics [M]. Beijing: Water Conservancy and Hydropower Press, 1983. (in Chinese)

[15] 王勖成．有限元法[M]．北京：清华大学出版社，2003.
Wang Xucheng. FEM [M]. Beijing: Tsinghua University Press, 2003. (in Chinese)

[16] 朱伯芳．有限单元法原理与应用[M]．北京：水利电力出版社，1979.
Zhu Bofang. The principle and application of FEM [M]. Beijing: Water Conservancy and Hydropower Press, 1979. (in Chinese)

[17] K.Washizu. Variational methods in elasticity and plasticity [M]. Pergamon Press Ltd, 1982.

[18] 卓家寿．弹塑性力学中的广义变分原理[M]．北京：水利水电出版社，1989.
Zhuo Jiashou. The principle of generalized variational principles in elasticity and plasticity [M]. Beijing: Water and Electricity Press, 1989. (in Chinese)

[19] 卓家寿．非线性固体力学基础[M]．北京：水利水电出版社，1996.
Zhuo Jiashou. Fundamentals of non-linear solid mechanics [M]. Beijing: Water Conservancy and Hydropower Press, 1996. in Chinese)

[20] D. R. J. Owen, E. Hinton. FEM in plasticity—Theory and Practice [M]. Pineridge Press Suanswa, UK, 1980.

[21] R.E.Goodman and G.H.Shi. Block theory and its application to rock engineering [M]. Prentice—Hall Inc, 1985.

[22] K. Kawai. New discrete structural models and generalization of the method of limit analysis [R]. Finite Element in Nonlinear Mechanics, NIT, Tronheim, 1977. 885~906.

[23] W. F. Chen and A. D. Pan. Finite Element and Finite Block Method in Geomechanics [J]. ASME Press, 1990. 669~675.

[24] Zhang Xiong and Qian Lingxi. Rigid finite element and limit analysis [J]. Acta Mechanica Sinica, 1993, 9(2): 156~162.

[25] 钱令希，张雄．结构分析中的刚体有限元法[J]．计算结构力学及应用，1991,2.
Qian Lingxi, Zhang Xong. Rigid body FEM in structure analysis [J]. Computational Structural Mechanics and Application, 1991, 2. (in Chinese)

[26] 卓家寿，赵宁．不连续介质静、动力分析的刚体-弹簧元法[J]．河海大学学报，1993, 21(5): 34~43.
Zhuo Jianshou, Zhao Ning. Rigid body-spring FEM of static and dynamic analysis in discontinuous medium [J]. J. of Hehai University, 1993, 21(5): 34~43. (in Chinese)

[27] Shi Genhua. Numerical manifold method [R]. Proc.Of IFDDA 96, Tsi Press, Berkeley Califorria, 1996. 52~204.

[28] 赵宁，卓家寿．结构的三维非线性界面元分析[J]．河海大学学报，1995, 23(2): 23~31.

Zhao Ning, Zhuo Jiashou. 3D nonlinear interface element analysis of structures [J]. J. of Hehai University, 1995, 23(2): 23~35. (in Chinese)
[29] 方义琳, 卓家寿, 章青. 具有任意形状单元离散模型的界面元法[J]. 工程力学, 1998, 15(2): 27~37.
Fang Yiling, Zhuo Jiashou, Zhang Qing. The interface element method in arbitrary shape elements [J]. Engineering Mechanics, 1998, 5(2): 27~37. (in Chinese)
[30] 卓家寿. 不连续介质力学问题的界面元法[M]. 北京: 科学出版社, 2000.
Zhuo Jiashou. Interface element method of discontinuous medium mechanics [M]. Beijing: Science Press, 2000. (in Chinese)
[31] M.M.Carroll. 固体力学研究的趋向和良机[J]. 力学进展, 1986, 16(4): 517~527.
M.M.Carroll. The tendency and good opportunity in solid mechanics [J]. Mechanics Progress, 1986, 16(4): 517~527. (in Chinese)
[32] J.L.Chaboche. Continuum damage mechanics, part I—general concepts [J]. J. of Applied Mechanics, 1988, 55: 59~63.
[33] L.M.Kachanov. Introduction to continuum damage mechanics [M]. Martinus Nijhoff Publishers, Dordrecht, 1986.
[34] L.E.Malvern. Introduction to the mechanics of a continuous medium [M]. Prentice—Hall, Inc., 1969.
[35] 黄克智. 非线性连续介质力学[M]. 北京: 清华大学出版社, 1989.
Hong Kezhi. Nonlinear continuous medium mechanics [M]. Beijing: Tsinghua University Press, 1989. (in Chinese)
[36] 匡振邦. 非线性连续介质力学基础[M]. 西安: 西安交通大学出版社, 1989.
Kuang Zhenbang. The basic of nonlinear continuous medium mechanics [M]. Xi'an: Xi'an Jiaotong University Press, 1989. (in Chinese)
[37] A.R. Ingraffea, W.H.Gerstle, P.Gergely. The fracture mechanics of bond in reinforced concrete [R]. Cornell University, Department of Structural, Report 81-10/655c, 1981.
[38] J.Lemaitre. A continuous damage mechanics model for ductile fracture [J]. J. of Eng. Materials and Tech., 107, Jan., 1985. 83~89.
[39] 李灏, 欧阳平. 损伤力学的进展[J]. 力学与生产建设, 1982. 100~104.
Li Hao, Ouyang Ping. Progress of damage mechanics [J]. Mechanics and Construction, 1982. 100~104. (in Chinese)
[40] 刘宝琛, 颜荣贵. 破裂岩石力学模型探讨[J]. 岩土工程学报, 1981, 3(4).
Liu Baochen, Yan Ronggui. Inquiry into model of broken rock mechanics [J]. J. Geotechnical Engineering, 1981, 3(4). (in Chinese)
[41] 余天庆. 混凝土的分段线性损伤模型[J]. 岩石混凝土断裂与强度, 1985, 2: 14~16.
Yu Tianqing. Sectional linear damage model of concrete [J]. Crack and Strength of Rock and Concrete, 1985,1. (in Chinese)
[42] 陈至达. 有理力学[M]. 北京: 中国矿业大学出版社, 1988.
Chen Zhida. Rational mechanics [M]. Beijing: Mining Industry University Press, 1988. (in Chinese)
[43] 王仁. 巷道大变形的粘性流体有限元分析[J]. 力学学报, 1985, 17(2).
Wang Ren. Stickness fluid FEM in the large deformation of lane [J]. Acta Mechanicas Sinica, 1985, 17(2). (in Chinese)
[44] 李灏. 损伤力学[M]. 武汉: 华中工学院出版社, 1986.
Li Hao. Damage mechanics [M]. Wuhan: Huazong Engrg. Inst. Press, 1986. (in Chinese)
[45] 谢和平, 陈至达. 岩石连续损伤力学模型探讨[J]. 煤炭学报, 1988, 1: 33~42.
Xie Heping, Chen Zhida. Model of rock continuous rock damage mechanics [J]. J. of Coal, 1988, 1: 33~42. (in Chinese)
[46] 谢和平. 非线性大变形边界元法[J]. 应用数学和力学, 1988, 9(12): 1087.
Xie Heping. Nonlinear large deformation BEM [J]. Applied Mathematics and Mechanics, 1988, 9(12): 1087. (in Chinese)
[47] 韩大建. 塑性、断裂及损伤在建立混凝土本构模型中的应用[J]. 力学与实践, 1988, 10(1).
Han Dajian. The application of plastic, fracture and damage mechanics in concrete structures [J]. Mechanics and Practice, 1988, 10(1). (in Chinese)
[48] S. Murakami. Notion of continuum damage mechanics and its application to anisotropic creep damdge theory [J]. J. Eng. Mater. and Tech., 1983, 105(2): 99~105.
[49] J.Lemaitre. How to use damage mechanics [J]. Nuclear Eng. and Design, 1984, 80: 233~245.
[50] C.Lee, F.A.Cozzarelli, K.Burke. One-dimensional strain-dependent creep damage in inhomogeneous materials [J]. Int. J. Nonlinear Mechanics, 1986, 21(4): 303~314.
[51] T.C.Chang, C.H.Popelar, G.H.A.Staab. A damage model for creep crack growth [J]. Int. J. of Fracture, 1987, 32: 157~168.
[52] M.F.Ashby. Creep fracture [R]. Proc.IUTAM Symp. Creep in Structure, 1980. 368.
[53] D.Krajcinovic. Creep of structure [J]. J. of Structure Mechanics, 1983: 11(1): 1~11.
[54] R.H.Martison, J.J.Hartog, G.C.Knollman. On the damage field near crack tips in a filled polymer [J]. Experimental Mechanics, 1982, 22(9): 329~335.
[55] F.A.Leckie, E.T.Onat. IUTAM Colloquium "Physical nonlinearities in structureal analysis" [R]. Berlin, 1981. 140~155.
[56] F.Sidoroff. IUTAM Colloquium "Physical nonlinearities in structureal analysis" [R]. Berlin, 1981. 237~244.
[57] J.Hult, J.Janson. Fructure mechanics and damage mechanics-A combined approach [J]. J. of Mechanics Application, 1977, 1: 69~84.
[58] D.Krajcinovic. Distributed damage theory of beam in pure bending [J]. J. Applied Mechanics, 1979, 46:

592~596.
[59] 徐绩善. 强度力论及应用[M]. 北京: 水利电力出版社, 1981.
Xu Jishan. Strength theory and its application [M]. Beijing: Water conservancy and Hydropower Press, 1981. (in Chinese)
[60] F.Sidoroff. IVTAM Colloquium “physical nonlinearit--ies in struc. analysis” [R]. Berlin, 1981. 237~244.
[61] J.A.Lemaitre. A continuous damage mechanics model for dructile fracture [J]. J. of Eng. Material & Technology, 107, 1985, 83~89.
[62] D.Krajcinovic. Constitutive equations for damage materials [J]. J. of Applied Mechanics, 50, 1983, 355~360.
[63] J.J.Marigo. Modelling of brittle and fatigue damage for elastic meterial by growth of microvoids [J]. Eng. Fracture Mechanics, 1985, 21(4): 861~874.
[64] H..D.Bui, A.Ehrlachar. The steady state propagation of a damaged zome of an elastic brittle [R]. Solid, ICF-6, 1984, 2.
[65] H.D.Bui. A.Ehrlachar. Propagation of damage in elastic and plastic solid [R]. Solid, ICF-5, 1983, 2: 86~89.
[66] R.Kamesware. Creep damage under stationary random loading [J]. J. App. Mechanics, Trans. Of ASME, 1977, 44: 763~764.
[67] D.Krajcinovic. Statistical aspects of the continuous damage theory [J]. Int. J. Solids Structures, 1982, 18(7): 551~562.
[68] G.C.Sih. Some basic problems in fracture mechanics [J]. Engrg. Fracture Mechanics, 1973, 5: 365~377.
[69] L.M.Kachanov. About crack growth problem under creep condition [J]. Probl. Mekh. Spl. Sredy, Moscow, 1969. (in Russion)
[70] K·Saanouni, J.L.Chaboche, C. Bathias. On the creep crack growth prediction by a local approach [J]. J. of Eng. Fracture, Mechanics, 1986, 25(516): 677~691.
[71] A.Charinteri, A.R.Ingraffea. Fracture Mechanics of Concrete: Material Characterization and Testing [M]. Martinus Nijhoff Publishers, 1984.
[72] J.Mazars. Mechanical damage and fracture of concrete structures [R]. ICF-5, 1983,4.
[73] S.Popovics. Fracture mechanism in concrete, How much do we know?[J]. Proc.of ASME, 95,EM3, 1969, 531~544.
[74] A.Carpinteri. Numerical modelling of damage and fracture in concrete[R]. ICF-6, 4, 1984.
[75] Z.P.Bazant, B·H·Oh. Concrete fracture via stress-strain relations[R]. Northwestern University, Report, 82-7, 1981.
[76] G.Frantziskonis, C.S.Desai. Constitutive model with strain softening [J]. Int. J. Solid Structure, 1987, 23(6): 733~768.
[77] H.L.Ewalds, R.J.H.Wanhill. Fracture machanics [M]. The Netherlands, E. Arnold Delftse Uitgevers Maatschappij, 1984, 226.
[78] A·H·Cottrell. Theory of brittle fracture in steel and similar materials [J]. Trans. AIME, 212, 1958, 192.
[79] 郑长卿. 韧性断裂细观力学的初步研究及应用[M]. 西安: 西北工业大学出版社, 1988.
Zheng Changqing. Preliminary study of tenacity fracture micro-mechanics and its application [M]. Xi’an: Northwest Industry University Press, 1988. (in Chinese)
[80] 杨卫. 损伤力学的细观理论[R]. 全国损伤力学讨论会, 1985.
Yang Wei. Micro-theory of damage mechanics [R]. Symposium on Damage Mechanise in China, 1985. (in Chinese)
[81] 王泳嘉, 邢继波. 离散单元发及其在岩土力学中的应用[M]. 沈阳: 东北工学院出版社, 1991.
Wang Yongjia, Xing Jibo. Dispersed element method and its application in rock-soil mechanics [M]. Shenyang: Northeast Technical University Press, 1991. (in Chinese)
[82] 朱以文. 大变形问题非连续变形分析[J]. 武汉水利电力大学学报, 1997.
Zhu Yiwen. Discontinuous deformation analysis of large deformation problems [J]. J. Wuhan Univ. of Water Conservancy and Hydropower, 1997. (in Chinese)
[83] 杨卫. 宏微观断裂力学[M]. 北京: 国防工业出版社, 1995.
Yang Wei. Macro-micro fracture mechanics [M]. Beijing: National Industry Press, 1995. (in Chinese)
[84] 卓家寿. 非线性固体力学基础[M]. 北京: 水利水电出版社, 1996.
Zhuo Jiashou. Fundamentals of nonlinear solid mechanics [M]. Beijing: Water-elestricity Press, 1996. (in Chinese)
[85] 章青, 卓家寿. 加錨岩体的界面元法[J]. 岩土工程学报, 1998, 20(5): 50~53.
Zhang Qing, Zhuo Jiashou. Stress element model of interface element method in anchor rock [J]. J. of Rock-soil Engineering, 1998, 20(5): 50~53. (in Chinese)
[86] 方义琳, 卓家寿. 传热场问题的界面元法[J]. 河海大学学报, 1998, 26(4): 87~91.
Fang Yilin, Zhuo Jiashou. Interface element method for heat conduct problems [J]. J. Hehai University, 1998, 26(4): 87~91. (in Chinese)
[87] 王卫标, 武清玺, 卓家寿. 二维问题的随机界面元法[J]. 河海大学学报, 1999, 27(4): 111~115.
Wang Weibiao, Wu Qingxi, Zhuo Jiashou. Random interface element method for 2D problems [J]. J. of Hehai University, 1999, 27(4): 111~115. (in Chinese)
[88] R.E.Goodman, R.L.Taylor, T.L.Brekke. A model of the mechanics of jointed rock [J]. J. of Soil Mechanics and Foundation Division, ASCE, ASM3, 1968, 94.
[89] C.S.Desai, B.K.Nagaraj. Modeling for cyclic normal & shear behavior of interfaces [J]. J. of Eng. Mechanics, ASCE, 1988, 114(EM7): 1198~1217.
[90] M.M.Zaman, C.S.Deasi, E.C.Drumm. Interface model for dynamic soil-structure interation [J]. J. of Geotechnical Eng., ASCE, 1984, 110(GT9): 1257~1273.
[91] G.H.Shi, R.E.Goodman. Discontinuous deforanation analysis, Proc. 25th US symposium on rock mechanics

[R]. 1984, 269~277.

[92] C.M.Gerrard. Elastic model of rock masses having one, two, three sets of joint [J]. Int. of Rock Mech. and Min. Eng. Sci. and Geomech Abstr, 1982, 19(1).

[93] P.A.Cundall. A computer model for simulating progressive large scale movements in blocky rock systems [R]. Proc. Symp. Rock Fracture (ISRM), Nancy, 1971.

[94] Te-chih Ke. The Issue of rigid-body rotation in DDA, DDA & simulations of discontinuous media [J]. Albuquerque, New Mexico, USA, 1996. 318~325.

[95] M.L.Kachanov. Microcrack model for rock inelasticity [D]. Thesis of Ph.D. of Rutgers Uni. Piscataway, 1980.

[96] Shi Gen-Hua. Simplex integration for manifold method [R]. DDA & analysis method Proc of IFDDA 96, Tsi Press, Berkeley, Califoria, USA ,1996, 206~260.

[97] M.Mary. Maclaughlin and Nicholas Sitar. Rigid body rotation in DDA, DDA and simulations of discontinuous Media [R]. Tsi Press, Albuquerque, New Mexico, USA, 1996, 620~635.

[98] T·Kawai, N.C.Chango. A discrete element analysis of beam bending problems in cluding the effects of shear deformation [J]. Seisan Venkyn, 1977, 29(4): 165~168.

[99] K.J.Bathe, S. Bolourchi. Large displacement analysis of 3D beam structures [J]. Int. J. Num. Meth. Eng., 1979, 14: 961~986.

[100] L.A.Givelli, C.A.Felippa. A 3D nonlinear Timoshenko bean based on the core-congruential formulation [J]. Int.J. Num. Meth. Eng., 1993, 36(21): 3647~3673.

[101] D.Ngo, A.C.Scordelic. FEM of Reinforced Concrete beams [J]. AOI J. 1967, 64(3): 152~163.

[102] 嵇醒, 臧跃拢, 程玉民. 边界元进展及通用程序[M]. 上海: 同济大学出版社, 1997.
Ji Xing, Zang Yuelong, Cheng Yumin. The progress and general program of BEM [M]. Shanghai: Tongji Univ. Press, 1997. (in Chinese)

[103] Belytschko T. Meshless method: An overview and recent developments [J]. Comp Methods Appl Mech Engrg, 1996, 139: 3~47.

[104] Li Shaofan, Liu W.K. Meshfree and particle methods and their applications [J]. Applied Mechanics Review, 2002, 55: 1~33.

[105] Lucy L B. A numerical approach to the testing of the fission hypothesis [J]. The Astron J, 1977, 8(12): 1013~1024.

[106] Lancaster P, Salkauskas K. Surfaces generated by moving least squaremethods [J]. Math Comput, 1981, 37: 141~158.

[107] Babuska I., Melenk J M. The partition of unity method [J]. Int. J Numer.Meth. Engrg, 1997, 40: 727~758.

[108] Liu W. K, Jun S, Zhang Y.F. Reproducing kernel particle methods [J]. Int J.Numer Methods Engrg, 1995, 20: 1081~1106.

[109] Nayroles B. Generalizing the finite element method: diffuse approximation and diffuse elements [J]. Comput Mech, 1992, 10: 307~318.

[110] Belytschko T. Element-free Galerkin methods [J]. Int J. Numer Methods Engrg, 1994, 37: 229~256.

[111] Duarte C. A, Oden J.T. Hp clouds-a meshless method to solve boundary-value problems [R]. Technical Report 95-05. Texas Institute for Computational and Applied Mathematics, University of Texas at Austin, 1995.

[112] Onarte E. A finite point method in computational mechanics [J]. Int. J. Numer. Methods Engrg., 1996, 39: 3839~3866.

[113] Atluri S. N., Zhu T.L. A new meshless local Petrov-Galerkin (MLPG) approach in computational mechanics [J]. Computational Mechanics, 1998, 22: 117~127.

[114] Atluri S.N., Zhu T.L. The meshless local Petrov-Galerkin (MLPG) approach for solving problemsin elasto-statics [J]. Computational Mechanics, 2000, 25: 169~179.

[115] Atluri S.N., Zhu T.L. New concepts in meshless methods[J]. International Journal for Numerical Method in Engineering, 2000, 47(1~3): 537~556.

[116] Atluri S.N., Kim H.G. A critical assessment of the truly meshless local Petrov-Galerkin (MLPG) and local boundary integral equation (LBIE) methods [J]. Computational Mechanics,1999, 24: 348~372.

[117] Liu W. K., Chen Y., Uras R. A., Chang C. T. Generalized multiple scale reproducing kernel particle [J]. Comput Methods Appl Mech Engrg, 1996, 139: 91~157.

[118] Liu W. K., Chen Y.J. Wavelet and multiple scale reproducing kernel methods [J]. Int. J. Numer. Methods Fluids, 1995, 21: 901~931.

[119] Chen W. New RBF Collocation methods and kernel RBF with applications [J]. In: Meshfree Methods for Partial Differential Equations (Griebel M and Schweitzer M A Eds.), Vol.1, Springer Verlag, 2000.

[120] Idelsohn S.R., Onate E., Calvo N., Del Pin F. The meshless finite element method [J]. Int J. Numer Methods Engrg, 2003, 58: 893~912.

[121] Hao S., Park H.S., Liu W.K. Moving particle finite element method [J]. Int. J. Numer Methods Eng., 2002, 53(8): 1937~1958.

[122] Atluri S.N., Sladek J. The local boundary integral equation (LBIE) and its meshless implementation for linear elasticity [J]. Computational Mechanics, 2000, 25: 1180~1198.

[123] Zhu T. Zhang J. D., Atluri SN. A local boundary integral equation (LBIE) method in computational mechanics, and a meshless discretization approach [J]. Computational Mechanics, 1998, 21: 223~235.

[124] Zhu Tulong, Zhang Jindong, Atluri SN. A meshless numerical method based on the local boundary integral equation (LBIE) to solve linear and non-linear boundary value problems [J]. Engineering Analysis With Boundary Elements, 1999, 23: 375~389.

[125] Zhu T., Zhang J.D., Atluri S.N. A meshless local boundary integral equation (LBIE) method for solving nonlinear problems [J]. Computational Mechanics, 1998, 22: 174~186.

[126] Mukherjee Y.X., Mukherjee S. The boundary node method for potential problems [J]. Internatioal Journal

for Numerical Methods in Engineering, 1997, 40: 797~815.

[127] Kothnur V.S., Mukherjee S., Mukherjee Y.X. Two dimensional linear elasticity by the boundary node method [J]. International Journal of Solids and Structures, 1999, 36: 1129~1147.

[128] Chati M.K., Mukherjee S., Mukherjee Y.X. The boundary node method for three-dimensional linear elasticity [J]. International Journal for Numerical Methods in Engineering, 1999, 46: 1163~1184.

[129] Mandar K.Chati and Subrata Mukherjee. The boundary node method for three-dimensional problems in potential theory [J]. Int. J. Numer. Meth. Engng., 2000, 47: 1523~1547.

[130] Chati M.K., Mukherjee S., Paulino G. H. The meshless hypersingular boundary node for three-dimensional potential theory and linear elastic problems [J]. Engineering Analysis With Boundary Elements, 2001, 25: 639~653.

[131] 程玉民, 陈美娟. 弹性力学一种边界积分无网格法[J]. 力学学报, 2003, 2, 181~186.
Cheng Yumin, Chen Meijuan. Meshless method of boundary integration in elasticity mechanics [J]. Acta Mechanica Sinica, 2003, 2: 181~186. (in Chinese)

[132] Swegle J.W., Hicks D.L., Attaway S.W. Smoothed particle hydrodynamics stability analysis [J]. J Compout Phys, 1995, 116: 123~134.

[133] Dyka C.T. Addressing tension instability in SPH methods [R]. Technical report NRL/MR/6384, NRL, 1994.

[134] Johnson G.R., Stryk R.A. Beissel S R. SPH for high velocity impact computations [J]. Comput Methods Appl Mech. Engrg., 1996, 139: 347~3730.

[135] Belytschko T., Gu L., Liu Y.Y. Fracture and crack growth by element-free Galerkin methods [J]. Model Simul Master Sci Engrg, 1994, 2: 519~534.

[136] Dolbow J., Belytschko T. Numerical integration of the Gaierkin weak form in meshfree methods [J]. Comput. Mech., 1999, 23: 219~230.

[137] Belytschko T., Krongauz Y. Smoothing and accelerated computations in the element free Galerkin method [J]. J. Comput. Appl. Math, 1996, 74: 111~126.

[138] Krongauz Y., Belytschko T. EFG approximation with discontinuous derivatives [J]. Int. J. Numer. Methods Engrg. 1998, 41: 1215~1233.

[139] Belytschko T., Gu L., Lu Y.Y. Fracture and crack growth by element free Galerkin methods [J]. Modlling Simul. Mater. Sci. Eng., 1994, 2: 519~534.

[140] Lu Y.Y., Belytschko T. Element-free Galerkin methods for wave propagation and dynamic fracture [J]. Comput. Methods Appl. Mech. Engrg, 1995, 126: 131~153.

[141] Ponthot J.P., Belytschko T. Arbitrary Lagrangian-Eulerian formulation for element-free Galerkin method [J]. Comput. Methods Appl. Mech. Engrg, 1998, 152: 19~46.

[142] Krysl P., Belytschko T. Analysis of thin shells by element-free Galerkin method [J]. Comput. Mech, 1995, 17: 26~35.

[143] Belytschko T., Lu Y.Y. Gu L., Tabbara M. Element-free Galerkin methods for static and dynamic fracture [J]. International Journal of Solids and Structures, 1995, 32: 2547~2570.

[144] Sukumar N., Moran B., Black T., Belytscbko T. An element-free Galerkin method for three-dimensional fracture mechanics [J]. Comput. Mech, 1997, 20: 170~175.

[145] Belytschko T., Organ D., Gerlach C. Element-free Galerkin methods for dynamic fracture in concrete [J]. Comput. Methods Appl. Mech. Engrg, 2000, 187: 385~399.

[146] Xu Y., Saigal S. An Element Free Galerkin analysis of steady dynamic growth of a mode I crack in elastic-plastic materials [J]. International Journal of Solids and Structures, 1999, 36: 1045~1079.

[147] Kargarmovin M.H, Toussi H.E, Faribotz S.J. Elasto-plastic element-free Galerkin method [J]. Comput. Mech, 2003, 33(3): 206-214.

[148] Bouillard Ph, Seleau S. Element-free Galerkin solutions for helmholtz problems: formulation and numerical assessment of the pollution effect [J]. Comput. Methods Appl. Mech. Engrg, 1998, 162: 317~335.

[149] Lee S.H, Yoon Y.C. An improved crack analysis technique by element-free Galerkin method [J]. International Journal for Numerical Methods in Engineering, 2003, 56: 1291~1314.

[150] Fleming M., Chu Y.A., Moran B., Belytschko T. Enriched element, free Galerkin methods for crack tip fields [J]. International Journal for Numerical Methods in Engineering, 1997, 40: 1483~1504 .

[151] Beissel S., Belytschko T. Nodal integration of the element free Galerkin method [J]. Comput. Methods Appl. Mech. Engrg, 1996, 139: 49~74.

[152] Smolinski P., Palmer T. Procedures for multi-time step integration of element free Galerkin methods for diffusion problems [J]. Comput. Struct, 2000, 77: 171~183.

[153] Mukherjee Y.X., Mukherjee S. On boundary conditions in the element-free Galerkin method [J]. Comput. Mechanics, 1997, 19: 264~270.

[154] Alves M.K, Rossi R. A modified element-free Galerkin method with essential boundary conditions enforced by an extended partition of unity finite element weight function [J]. International Journal for Numerical Methods in Engineering 2003, 57: 1523~1552.

[155] Kaljevic I., Saigal S. An improved element free Galerkin formulation [J]. International Journal for Numerical Methods in Engineering, 1997, 40: 2953~2974.

[156] Krysl P., Belytschko T. ESFLIB: A library to compute the element free Galerkin shalSe functions [J]. Comput. Methods Appl. Mech. Engrg, 2001, 190: 2181~2205.

[157] Krysl P., Belytschko T. Element-free Galerkin method convergence of the continuous and discontinuous shape functions [J]. Comput. Methods Appl. Mech. Engrg, 1997, 148: 257~277.

[158] Gavete L., Falcon S., Ruiz A. An error indicator for the

element free Galerkin method [J]. Eur. J. mech. A/Solids, 2001, 20: 327~341.
[159] Duarte C.A, Oden J.T. An h-p adaptive method using clouds [J]. Comput. Meth. Appl. Mech. Engrg, 1996, 139: 237~262.
[160] Babuska I., Melenk J. M. The partition of unity finite element method: basic theory and applications [J]. Comput. Meth. Appl. Mech. Engrg, 1996, 139: 289~314.
[161] Chen J.S., Pan C.,Wu C.T., Liu W.K. Reproducing kernel particle methods for large deformation analysis of non-linear structures [J]. Computer Meth. Appl. Mech. Engrg, 1996, 139: 195~227.
[162] Li S., Liu W.K. Moving least-square reproducing kernel method (I) Methodology and Convergence [J]. Comput. Meth. Appl. Mech. Engrg, 1997, 143: 113~154.
[163] Li S.,Liu W.K. Moving least-square reproducing kernel method Part II Fourier analysis [J]. Comput. Meth. Appl. Mech. Engrg, 1996, 139: 159~193.
[164] Chen J.S., Yoon S., Wang H.P.Liu W.K. An improved reproducing kernel particle method for nearly incompressible finite elasticity [J]. Comput. Meth. Appl. Mech. Engrg, 2000, 181: 117~145.
[165] Voth T.E., Christon M. A. Discretization errors associated with reproducing kernel methods dimensional domains [J]. Comput. Meth. Appl. Mech. Engrg, 2001, 190: 2429~2446.
[166] Liew K., Ng T., Wu Y. Meshfree method for large deformation analysis-A reproducing kernel partical approach [J]. Engrg. Structure, 2000, 24: 543~551.
[167] Shangwu X., Liu W.K., Cao J. On the utilzation of the reproducing kernel particle method for the numerical simulation of plane strain rolling [J]. Int. Journal of Machine Tools and Manufacture, 2003, 43: 89~102.
[168] Chen J.S., You Y., Meng X. A reproducing kernel method with nodal interpolation property [J]. International Journal for Numerical Methods in Engineering, 2003, 56: 935~960.
[169] Hah W, Meng X. Error analysis of the reproducing kernel particle method [J]. Comput. Meth. Appl. Mech. Engrg, 2001, 190: 6157~6181.
[170] Onarte E, Idelsohn S. A mesh-free finite point method for advective-diffusive transport and fluid flow problems [J]. Comput. Mech., 1998, 21: 283~292.
[171] Buhmann M.D. Radial Basis Functions [J]. Acta Numerica, 2000, 1~38.
[172] Franke C., Schaback S. Convergence order estimates of meshless collocation methods using radial basis functions [J]. Advances in Computational Mathematics, 1998, 8: 381~399.
[173] Wu Z.M, Hon Y.C. Convergence error estimate in solving free boundary diffusion problem by radial basis functions method[J]. Engineering Analysis with Boundary Elements, 2003, 27: 73~79.
[174] Wang J.G., Liu G.R. On the optimal shape parameters of radial basis functions used for 2-D meshless methods [J]. Comp. Meth. Appl. Mech. Engrg, 2002, 191: 2611~2630.
[175] Wang J., Liu G. A point interpolation meshless method based on radial basis functions [J]. Int. Journal for Num. Methods in Engineering, 2002, 54: 1623~1648.
[176] Zhang Jianming, Yao Zhenhan, Li Hong. A hybrid boundary node method [J]. Int. Journal for Num. Methods in Engrg., 2002, 53: 51~763.
[177] Belytschko T, Organ D. Coupled finite element-element-free Galerkin method [J]. Comp. Mech. 1995, 17: 86~195.
[178] Hegen D. Element-free Galerkin methods in combination with finite element approaches [J]. Com. Method Appl. Mech. Engrg., 1996, 135: 143~166.
[179] Liu GR, Gu YT. Coupling of element-free Galerkin and hybrid boundary element methods using modified variational formulation [J]. Comp. Mech. 2000, 26(2): 166~173.
[180] Liu G R, Gu Y. Meshless local Petrov-Galerkin (MLPG) method in combination with finite element and boundary element approaches [J]. Comp. Mechanics, 2000, 26: 536~546.
[181] Antonio Huerta, Sonia Fernadez-Mendez, Enrichment and coupling of the finite element and meshless methods [J]. Int. Journal Num. Methods in Engrg., 2000, 48: 1615~1636.
[182] Gu Y., Liu G.R. A coupled element-free Galerkin /boundary element method for stress analysis of two-dimension solid [J]. Comp. Method in Appl. Mech. Eng., 2001, 190: 4405~4419.
[183] Rao B., Rahman S. A coupled meshless-finite element method for fracture analysis [J]. Int. Journal of Pressur Vessel & Piping, 2001, 78: 647~657.
[184] Xiao Q., Dhanasekar M. Coupling of FE and EFG using collocation approach [J]. Adv. in Engrg. Software, 2002, 33: 507~515.
[185] Chen T, Raju I. A coupled finite element and meshless local Petrov-Galerkin method for two-dimensional potential problems [J]. Comp. Meth. Appl. Mech. Engrg, 2003, 192: 4533~4550.
[186] 宋祖康, 陆明万, 张雄. 固体力饿中的无网格方法[J], 力学进展, 2000, 30(1): 55~65.
Song Zukang, Lu Mingwan, Zhang Xiong. Meshness method in solid mechanics [J]. Mechanics Progress, 2000, 30(1): 55~65. (in Chinese)
[187] 张雄, 宋祖康, 陆名万. 无网格法研究新进展及其应用[J]. 计算力学学报, 2003, 20(6): 731~741.
Zhang Xong, Song Zukang, Lu Mingwan. The progress and application in the study of meshless method [J]. J. Compute Mechanics, 2003, 20(6): 731~741. (in Chonese)
[188] 姜宏道, 曹国金. 无单元法研究和应用现状及动态[J]. 力学进展, 2002, 34(4): 526~534.
Jiang Hongdao, Cao Guojin. Current situation and trends in meshless study and application [J]. Mechanics Progress, 2002, 34(4): 526~534. (in Chinese)
[189] 张锁春. 光滑质点流体动力学(SPH)方法[J]. 计算物理, 1996, 13(4): 385~397.
Zhang Suochun. The method of smoothed partical hydrodynamics [J]. Compute Physics, 1996, 13(4):

385~397. (in Chinese)
[190] 贝新源, 岳钟五. 三维SPH程序及其在斜高速碰撞问题的应用[J]. 计算物理, 1997, 14(2): 155~166.
Bei Xinyuan, Yue Zhongwu. 3D SPH program and its application in high speed collision problems [J]. Comput. Physics, 1997, 14(2): 155~166. (in Chinese)
[191] Liu X, Lu M.W, Zhang X. Numerical analysis of singular problems using the partition of unity method[J]. European Comp. Mechanics (ECCM'99). Munchen, Germany, 1999.
[192] Song K.Z., Zhang X. Meshless method based on collocation for elasto-plastic analysis [J]. Proceedings of Internal conference on computational engineering & science.August, 2000. 20~25.
[193] Zhang X., Song K.Z., Lu M.W. Meshless methods based on collocation with radial basis function [J]. Comp. Mech., 2000, 26(4): 333~343.
[194] Xiong Zhang, Xiao-hu Liu, Kang-zu Song and Ming-wan Lu. Least-squares collocation meshless method [J]. Int. J. Numer. Meth. Engrg., 2001, 51: 1089~1100.
[195] 李梅娥, 张陵. 再生核质点法研究进展[J]. 力学进展, 2002, 35(4): 535~544.
Li Mei'e, Zhang Ling. The progress of reproducing kernel partical method [J]. Mechanics Progress, 2002, 32(4): 535~544. (in Chinese)
[196] 张雄, 胡煒. 加权最小二乘无网格法[J]. 力学学报, 2003, 35(4): 425~431.
Zhang Xong, Hu Wei. Weighted least-square meshless method [J]. Acta Mechanica Sinica, 2003, 35(4): 425~431. (in Chinese)
[197] 张雄, 宋祖康, 陆明万. 紧支试函数加权残余法[J]. 力学学报, 2003, 35(1): 43~49.
Zhang Xong, Song Zukang, Lu Mingwan. Weighted residual method of radial trial function [J]. Acta Mechanica Sinica, 2003, 35(1): 43~49. (in Chinese)
[198] 娄路亮, 曾攀. 影响无网格法求解精度的因素分析[J]. 计算力学学报, 2003, 20(3): 313~319.
Lou Luliang, Zeng Pan. Factor analysis of solution precision in meshless method [J]. Comput. Mechanics, 2003, 20(3): 313~319. (in Chinese)
[199] 蔡永昌, 朱合华. 基于 Voronoi 结构的无网格局部 Petro-Galerkin 方法[J]. 力学学报, 2003, 35(2): 187~193.
Cai Yongchang, Zhu Hehua. Meshless local Petrov-Galerkin method based on the Voronoi structure [J]. Acta Mechanica Sinica, 2003, 35(2): 187~193. (in Chinese)
[200] 蔡永昌, 朱合华. 岩土工程数值计算中的无网格法及其全自动布点技术[J]. 岩土力学, 2003, 24(1): 21~24.
Cai Yongchang, Zhu Hehua. Meshless method and automatic set point technique for numerical calculation in rock-soil engineering [J]. J. Rock and Geotechnics, 2003, 24(1): 21~24. (in Chinese)
[201] 龙述尧, 许敬晓. 弹性力学问题的局部积分方程方法[J]. 力学学报, 2000, 32(5): 566~577.
Long Shuyao, Xu Jingxiao. Local integral equition method in elasticity mechanics [J]. Acta Mechanica Sinica, 2000, 32(5): 566~577. (in Chinese)
[202] Zhang X. A 2D meshless model for jointed rock structures [J]. Int. J. Numer. Methods Engrg., 2000, 47(10): 1649~1661.
[203] 寇晓东, 周维垣. 应用无单元法近似计算拱坝开裂[J]. 水利学报, 2000, 10: 28~35.
Kou Xiaodong, Zhou Weiyuan. The approximate computation in crack of arch dams using element free methods [J]. J. Water Conservancy, 2000, 10: 28~35. (in Chinese)
[204] 张伟星, 庞辉. 弹性地基板计算的无单元法[J]. 工程力学, 2000, 17(3): 138~144.
Zhang Weixing, Pang Hui. Element free method for plates on elasticity foundation [J]. Engineering Mechanics, 2000, 17(3): 138~144. (in Chinese)
[205] 庞作会, 葛修润. 无网格 Galerkin 法在边坡开挖中的应用[J]. 岩土力学, 1999, 20(1): 61~64.
Pang Zuohui, Ge Xiuren. The application of meshless Galerkin method in slope excavation [J]. J. Rock and Geotechnics, 1999, 20(1): 61~64. (in Chinese)
[206] 陈建, 吴林志, 杜善义. 采用无单元法计算含边缘裂纹功能梯度材料板的应力强度因子[J]. 工程力学, 2000, 17(5): 139~144.
Chen Jian, Wu linzhi, Du Shanyi. The calculation of stress intensity factor using element free method along single crack material plate [J]. Engineering Mechanics, 2000, 17(5): 139~144. (in Chinese)
[207] 曾清红, 卢德唐. 无网格法求解稳定渗流问题[J]. 计算力学学报, 2003, 20(4): 440~445.
Zeng Qinghong, Lou Detang. Meshless method solving stable permeateion problem [J]. J. Comp. Mechanics, 2003, 20(4): 440~445. (in Chinese)
[208] 郭隽, 陶智. 非线性问题的MPS无网格算法[J]. 北京航空航天大学学报, 2003, 29(1): 83~86.
Guo Juan, Tao Zhi. MPS meshness method for nonlinear problems [J]. J. Beijing Aerospace and Astronautics University, 2003, 29(1): 83~86. (in Chinese)
[209] 张延军, 王思敬. FEM与EFGM耦合法分析二相连续多孔介质[J]. 计算物理, 2003, 20(2): 142~146.
Zhang Yanjun, Wang Sijing. Coupled FEM and EFGM method for 2D continuous porous medium [J]. Comput. Physics, 2003, 20(2): 142~146. (in Chinese)
[210] 娄路亮, 曾攀. 应力高梯度问题的无网格分析[J]. 应用力学学报, 2002, 19(2): 121~124
Lou Luliang, Zeng Pan. Meshness analysis in high stress gradient problem [J]. J. Applied Mechanics, 2002, 19(2): 121~124. (in Chinese)
[211] 李振, 李子然. EFGM 模拟复合型疲劳裂纹的扩展[J]. 工程力学, 2002, 19(1): 25~28.
Yuan Zhen, Li Ziran. Simulation of expanded tired crack with meshless method [J]. Engrg. Mech., 2002, 19(1): 25~28. (in Chinese)
[212] 龙述尧. 用无网格局部 Petrov-Galerkin 法分析非线性弹性地基梁[J]. 力学季刊, 2002, 23(4): 547~551.
Long Shuyao. Analysis of beams on nonlinear foundation using meshless local Petrov-Galerkin method [J]. Quarter J. Mechanics, 2002, 23(4): 547~551.

(in Chinese)

[213] 李卧东, 陈晓波. 无网格法在弹塑性问题中的应用[J]. 固体力学学报, 2001, 22(4): 361~367.
Li Wodong, Chen Xiaobo. The application of meshless method in elasticity-plasticity problems [J]. Acta Mechanica Solida Sinica, 2001, 22(4): 361~367. (in Chinese)

[214] 李卧东, 陈晓波. 无网格法在断裂力学中的应用[J]. 岩石力学与工程学报, 2001, 20(4): 462~466.
Li Wodong, Chen Xiaobo. The application of meshless method in fracture mechanics [J]. J. Rock Mechanics and Engineering, 2001, 20(4): 462~466. (in Chinese)

[215] 庞作会, 朱岳明. EFGM 求解接触问题[J]. 河海大学学报: 自然科学版, 2000, 28(4): 54~58.
Pang Zuohui, Zu Xueming. EFGM for solving contact problems [J]. J. Hehai University, 2000, 28(4): 54~58. (in Chinese)

[216] 庞作会, 葛修润. EFGM 模拟不连续面[J]. 工程地质学报, 2000, 8(3): 364~368.
Pang Zuohui, GeXiuren. EFGM for simulating discontinuous surface [J]. J. Engrg. Geology, 2000, 8(3): 364~368. (in Chinese)

[217] 庞作会, 葛修润. EFGM 的两点补充[J]. 岩石力学与工程学报, 1999, 18(5): 581~584.
Pang Zuohui, Ge Xiuren. Two complements about EFGM [J]. J. Rock Mechanics and Engrg., 1999, 18(5): 581~584. (in Chinese)

[218] 夏道行, 吴作人. 实变函数论与泛函分析[M]. 北京: 高等教育出版社, 1979.
Xia Daohang, Wu Zuoren. General functional analysis [M]. Beijing: Higher Education Press, 1979. (in Chinese)

[219] Timoshenko S.P., Goodier J.N. Theory of elasticity [M]. (Third edition), McGraw-Hill Inc., 1970.

[220] Rao B.N., Rahman S. An enriched meshless method for nonlinear fracture mechanics [J]. International Journal for Numerical Methods in Engineering, 2004, 59: 1~27.

[221] Dolbow J. An extended FEM with discontinuous enrichment for applied mechanics [D]. Ph.D thesis of Northwest University, USA, 1999.

[222] Daux C., Moes N., Dolbow J., Sukumar N., Belytschko T. Arbitrary branched and intersecting cracks with the extended finite element method [J]. Int. J. Numer. Meth. Engrg, 2000, 48: 1741~1760.

[223] Dolbow J., Moes N., Belytschko T. An extended finite element method for modeling crack growth with frictional contact [J]. Comput. Meth. Appl Mech Engrg, 2001, 190: 6825~6846.

[224] Anderson T.L. Fracture mechanics: Fundamentals and applications [M]. (First Ed.). CRC Press, 1999.

[225] 程玉民, 沈祖炎, 彭妙娟. 加权残数与有限元耦合发解弹性力学问题[J]. 土木工程学报, 2000, 33(4).
Cheng Yumin, Shen Zuyan, Peng Miaojuan. Weighted residual and FEM for solving coupled elasticity problems [J]. J. Civil Engrg., 2000, 33(4). (in Chinese)

[226] Cheng Yumin, Peng Miaojuan. The coupling method of boundary and finite elements for a nonlinear problem of structural engineering [M]. in: Theory and Applications of Boundary Element Methods, (Zhenhan Yao and M. Tanaka Eds.), International Academic Publishers, 1998.

[227] T.Von Karman. The Engineer Grapples with Nonlinear Problems [R]. Bulletin of the American Mathematical Society, 1940, 46(8): 615~683.

[228] Bathe K.J. and Wilson E.L. Numerical methods in finite element analysis [M]. Prentice- Hall, Inc, 1976.

[229] Zienkiewicz O.C. and Morgan K. Finite element and approximation [M]. John Wiley & Sons, Inc, 1983.

[230] Owen D.R.J, Hinton E. Finite Element in Plasticity, Theory and Pratice [M]. Pineridge Press, 1982.

[231] 殷有泉. 固体力学非线性有限元引论[M]. 北京: 北京大学出版社, 1986.
Yin Youquan. Introduction of nonlinear FEM in solid mechanics [M]. Beijing: Peking University Press, 1986. (in Chinese)

[232] 王宏纲. 热弹性[M]. 北京: 清华大学出版社, 1986.
Wang Honggang. Hot elasticity [M]. Beijing: Tsinghua University Press, 1986. (in Chinese)

[233] 周维垣. 高等岩石力学[M]. 北京: 水利水电出版社, 1990.
Zhou Weiyuan. Higher rock mechanics [M]. Beijing: Water Conservancy and Hydropower Press, 1990. (in Chinese)

[234] 董哲仁. 钢筋混凝土非线性有限元原理及应用[M]. 北京: 中国铁道出版社, 1993.
Dong Zheren. Principle and application of nonlinear FEM in reinforced concrete [M]. Beijing: Railway Press, 1993. (in Chinese)

[235] 董毓利. 混凝土非线性力学基础[M]. 北京: 建筑工业出版社, 1997.
Dong Yuli. Foundation of concrete nonlinear mechanics [M]. Beijing: Architecture Industry Press, 1997. (in Chinese)

[236] 江见鲸. 钢筋混凝土非线性有限元分析[M]. 陕西: 陕西科技出版社, 1994.
Jiang Jianjing. Nonlinear FEM of reinforce concrete [M]. Shaanxi: Shaanxi Science and Technology Press, 1994. (in Chinese)

[237] 殷有泉, 张宏. 岩土系统弹塑性应力应变分析稳定性分析的 NOLM 程序[C]. 北京: 北京大学出版社, 1985.
Yin Youquan, Zhang Hong. NOLM program of stress deformation and stability analysis in elasticity and plasticity of rock-soil system [C]. Beijing: Peking University Press, 1985. (in Chinese)

[238] Dragon A. and Mroz Z. A continuum model for plastic-brittle behaviour of concrete [J]. Int. J. Engineering Science, 1979, 17: 121~137.

[239] 王仁, 梁北援. 巷道大变形的粘性流体有限元分析[J]. 力学学报, 1985, 17.
Wang Ren, Liang Beiyuan. Viscosity fluid FEM of large deformation of tunnel [J]. Acta Mechanica Sinica, 1985, 17. (in Chinese)

[240] 兰腊保, 王仁. 倒转褶皱的弹塑性有限元模型[C]. 北京: 北京大学出版社, 1985.
Lan Labao, Wang Ren. Inverted fold FEM model of elasticity and plasticity [C]. Beijing: Peking University

Press, 1985. (in Chinese)
[241] 杨菊生, 揽生瑞, 李九红. 混凝土三轴非线性比例加载试验与结构开裂分析[C]. 结构工程与振动研究报告集, 清华大学出版社, 2000. 23~35.
Yang Jusheng, Lan Shengrui, Li Jiuhong. Tri-axial nonlinear loading experinment of concrete and structural crack analysis [C]. Thesis of Structural Engineering and Vibration, Tsinghua University Press, 2000. 23~35. (in Chinese)
[242] 杨菊生, 李九红, 郭永重. 混凝土结构非线性损伤有限元分析[C]. 结构工程与振动研究报告集, 清华大学出版社, 2000. 36~45.
Yang Jusheng, Li Jiuhong, Guo Yongzhong. Study of finite element analysis of non-elastcity damage of concrete structure [C]. Collected Works on Structural Engineering and Vibrations, Tsinghua Univ. Press., 2000. 36~45. (in Chinese)
[243] Li Jiuhong, Lan Shengrui, Yang Jusheng. Three-Dimensional Nonlinear FEA under Cracking of Concrete Structure [J]. International Symposium on New Development in Concrete Science and Technology, Nanjing, 1995.
[244] Lan Sheng, Yang Jusheng. The space structural of blast furnace [R]. International Conference on Mechanics of Solids and Materials Engineering, Singaper, 1995.
[245] 许汉铮, 杨菊生. 混凝土结构三维弹塑性开裂分析[J]. 工程力学, 1996, 1: 400~404.
Xu Hanzheng, Yang Jusheng. 3D elasticity-plastucity crack analysis of concrete structures [J]. Engrg. Mechanics, 1996, 1: 400~404. (in Chinese)
[246] Yang Jusheng, Li Jiuhong, Guo Yongzhong. The damage cracking model of brittle material in numerical analysis [R]. Computer Method and Advances in Geomechanics, 3, Ninth International Conference of The Association for Computer Method and Advances in Geomechanics, 1997.
[247] Lan Shengrui, Yang Jusheng. Nonlinear FEM of arch dams Ⅰ:Constitutive relationship [J]. Advances in Engineering Software, 1997, 28.
[248] Lan Sheng, Yang Jusheng. Nonlinear FEM of arch dams 一Ⅱ: Nonlinear analysis [J]. Advances in Engineering Software, 1997, 28.
[249] 杨菊生, 李九红, 刘华. 脆性材料损伤-破坏耦合模型[J]. 岩石力学与工程学报, 1998, 17: 749~754
Yang Jusheng, Li Jiuhong, Liu Hua. Damage-failure coupling model of brittle material [J]. J. Rock Mechanics and Engineering, 1998, 17: 749~754. (in Chinese)
[250] 李九红, 杨菊生. 岩体损伤-弹塑性非线性有限元分析[J]. 岩石力学与工程学报, 1999, 19(1): 707~711.
Li Jiuhong, Yang Jusheng. Nonlinear finite element analysis for elasticity-plasticity of rock body [J]. J. Rock Mechanics and Engineering, 1999, 19(1): 707~711. (in Chinese)
[251] Balmer G.G. Shearing strength of concrete under high triaxial stress－computation of Mohr's envelope as a curve [R]. Bur. Reclam. Struct. Res. Lab. Rep. Sp－23, 1949.
[252] Drucker D.C. A definition of stable inelastic material [J]. J. Appl. Mech, 26, ASME Trans, 81, 1995.
[253] Bresler B, Pister K.S. Strenghth of concrete under combined stress [J]. J. Am. Coner. Inst, 1958, 55.
[254] Willam K.J, Warnke E.P. Constitutive model for the triaxial behavior of concrete [J]. Int. Assoc. Eng. Sem, 1974.
[255] Ottosen N.S. Failure and elasticity of concrete [R]. Dan. Atom. Energy Comm. Res. Estab. Ris Eng Dept. Ris－M－1801, Roskilole, Denmark, July, 1975.
[256] Hsieh S. S, Ting E.C, Chen W.F. An elastic fracture model for concrete [R]. Proc.3d Eng. Mech. Div. Spec. Conf. ASCE., Austin, Tex, 1979.
[257] 过镇海, 王传志. 多轴应力下混凝土的强度和破坏准则研究[J]. 土木工程学报, 1991, 3.
Guo Zhenhai, Wang Chuanzhi. Study of intensity and failure criteria of concrete under multi-axial stress [J]. J. Civil Engineering, 1991, 3. (in Chinese)
[258] 俞茂宏. 双剪应力强度理论研究[M]. 西安: 西安交通大学出版社, 1988.
Yu Mohong. Study of twin-shear stress intensity [M]. Xi'an: Xi'an Jiaotong University Press, 1988. (in Chinese)
[259] 俞茂宏. 双剪理论及其应用[M]. 北京: 科学出版社, 1998.
Yu Mohong. Twin-shear theory and application [M]. Beijing: Science Press, 1998. (in Chinese)
[260] 江见鲸, 贺小岗. 混凝土本构关系的现状及展望[J]. 工程力学, 1993.
Jiang Jianjing, He Xiaogang. Present situation and prospects of constitutive relationship of concrete [J]. Engrg. Mechanics, 1993. (in Chinese)
[261] Darwin D, Pecknold D.A. Nonlinear biaxial stress strain law for concrete [J]. ASCE 103(EM2), April, 1997.
[262] Kupfkr H, Gerstle K.H. Behavior of concrete under biaxial stresses [J]. ASCE, 99(EM4), Aug. 1973.
[263] Ottosen N.S. Constitutive model for short-time loading of concrete [J]. ASCE, 105(EM1), 1997.
[264] Gerstle K. Simple formulation of biaxial cncrete behavior [J]. I. J. Jan, Feb. 1981.
[265] Stankowski T., Gerstle K. Simple formulation of concrete behavior under multiaxial load histories [J]. ACIJ, March-april, 1985.
[266] Chen W.F. Plasticity in Reinforced Concrete [M]. Mc-Graw-hill, 1982.
[267] Hsieh S S, Ting E C and Chen W F. A plastic－fracture model for concrete [J]. Int. J. Solids Structure, 1982, 18(3).
[268] Hen D J and Chen W F. A no uniform hardening plasticity model for concrete material [J]. Mach. of Materials, 1985, 4.
[269] Vermeer P A and Borst R. Non－associated plasticity for soils [J].Concrete & rock, Heron , ACIJ. March, 1976.
[270] 董毓串, 谢和平. 高压混凝土损伤结构模型[J]. 力学与实践, 1996, 6.
Dong Yuchuan, Xie Heping. Damage structure model of higher pressure concrete [J]. Mechanics and Practice,

1996, 6. (in Chinese)

[271] Valanis K C. Theory of viscoplasticity without a yield surface [J]. Archiwum Mech. Stocssowanej, 1971, 23.

[272] Bazant E.P, Bhat D.P. Endochronic theory of inelasticity and failure of concrete [J]. ASCE, 102(EM4), Aug, 1976.

[273] 叶献国. 三轴高压混凝土强度子变形试验研究[D]. 北京: 清华大学, 1988.
Ye Xianguo. Deformed experiment study of concrete intensity factor under tri-axial higher pressure [D]. Beijing: Tsinghua University, 1988. (in Chinese)

[274] Lan Shengrui, Ynag Jusheng. Regional cracking model in FEA of reinforced concrete [J]. Int. Symposium of Concrete Engineering (Nanjing, China), 1991, 3.

[275] 李九红. 混凝土三轴试验与允许开裂的三维非线性有限元分析[D]. 西安: 西安理工大学, 1994.
Li Jiuhong. Tri-axial experiment of concrete and the 3D nonlinear FEM permiting crack [D]. Xi'an: Xi'an Univ. of Tcch., 1994. (in Chinese)

[276] 李九红. 复变量无网格法及其应用研究[D]. 西安: 西安理工大学, 2004.
Li Jiuhong. On meshless method with complex variables and applivations [D]. Xi'an: Xi'an Univ. of Tech., 2004. (in Chinese)

[277] 赵钦. 无网格法应用及影响求解精度的研究[D]. 西安: 西安理工大学, 2005.
Zhao Qin. On meshless method application and influences of solution precision [D]. Xi'an: Xi'an Univ. of Tech., 2005. (in Chinese)

[278] 寇晓东, 周维垣. 应用无单元法追踪裂纹开展[J]. 岩石力学与工程学报, 2000, 19(1): 18~23.
Kou Xiaodong, Zhou Weiyuan. Tracking crack expansion by element free method [J]. J. Rock-soil Mechanics and Engrg., 2000, 19(1): 18~23. (in Chinese)

[279] 周维垣, 杨若琼, 郯公瑞. 流形元法及其在工程中的应用[J]. 岩石力学与工程学报, 1996, 15(3): 211~218.
Zhou Weiyuan, Yang Roqong, Tan Gongrui. Manifold method and its application in engineering [J]. J. Rock Mechanics and Engineering, 1996, 15(3): 211~218. (in Chinese)

[280] 周维垣, 寇晓东. 无单元法及其应用[J]. 岩石力学与工程学报, 1998, 30(2): 193~202.
Zhou Weiyuan, Kou Xiaodong. Element free method and its application [J]. J. Rock Mechanics and Engrg, 1998, 30(2): 193~202. (in Chinese)

[281] 寇晓东, 杨若琼, 沈大利, 周维垣. 无单元法在小湾拱坝开裂计算中的应用[J]. 云南水力发电, 2000, 16(1): 71~76.
Kou Xiaodong, Yang Ruoqong, Shen Dali, Zhou Weiyuan. The application in crack calculation of Xiaowan Arch Dams using element free methods [J]. Yunnan Hydropower, 2000, 16(1): 71~76. (in Chinese)

[282] 张延军, 肖树芳. EFGM--在岩土工程上有限元法的有力补充[J]. 计算力学学报, 2003, 20(2): 179~183.
Zhang Yanjun, Xiao Shufang. Element free method (EFGM)—Powerful complement of FEM in rock-soil engineering [J]. J. Comput. Mechanics, 2003, 20(2): 179~183. (in Chinese)

[283] 杨菊生, 揽生瑞. 有限元法程序设计[M]. 西安: 西安交通大学出版社, 1991.
Yang Jusheng, Lan Shengrui. Programing of FEM [M]. Xi'an: Xi'an Jiaotong University Press, 1991. (in Chinese)

[284] 王国庆. 无单元法与有限元耦合的研究进展[J]. 浙江水电学校学报, 2003, 15(3): 4~6.
Wang Guoqing. Progress of study of coupled element free and FEM [J]. J. Zhejiang School of Water and Electricity, 2003, 15(3): 4~6.6(in Chinese)

[285] 何沛祥, 李子然, 吴长春. EFGM 与 FEM 的耦合及其对功能梯度材料断裂计算的研究[J]. 中国科技大学学报, 2001, 31(6): 672~680.
He Peixiang, Li Ziran, Wu Changchun. The coupled FEM - EFGM and its application in analysis of material crack [J]. J. Technology University of China, 2001, 31(6): 672~680. (in Chinese)

[286] 刘天祥, 刘更, 徐华. 二维 EFGM-FEM 耦合方法的研究[J]. 机械强度, 2002, 24(4): 602~607.
Liu Tianxiang, Liu Geng, Xu Hua.The study of 2D EFGM—FEM coupling methods [J]. Mechanical Strength, 2002, 24(4): 602~607. (in Chinese)

[287] 刘天祥, 刘更. 二维接触问题的 EFGM-FEM 耦合方法[J]. 西北工业大学学报, 2003, 21(4): 499~503.
Liu Tianxiang, Liu Geng. EFGM and FEM coupling method of two-dimensional contact problems [J]. J. Northwest Industry University, 2003, 21(4): 499~503. (in Chinese)

[288] 杨海天, 刘岩. 一种 FEM-EFGM 耦合技术及其应用[J]. 计算力学学报, 2003, 20(5): 511~517.
Yang Haitian, Liu Yan. An FEM–EFGM coupling technique and its application [J] J. Rock Mechanics and Engineering, 2003, 20(5): 511~517. (in Chinese)

[289] 张延军. EFGM-FEM 耦合法分析砂井地基的固结变形[J]. 岩石力学与工程学报, 2003, 22(9): 1542~1546.
Zhang Yanjun. FEM-EFGM coupling method for the analysis of reinforcement deformation of sand foundation [J]. J. Rock Mechanics and Engineering, 2003, 22(9): 1542~1546. (in Chinese)

[290] 张延军. EFGM-FEM 耦合法在固结方程中的应用及粘塑性蠕变分析[J]. 岩石力学与工程学报, 2003, 21(8): 1398~1402.
Zhang Yanjun. FEM and EFGM coupling method in the application of reinforcement deformation equation and the stickiness plasticity creep analysis [J]. J. Rock Mechanics and Engineering, 2003, 21(8): 1398~1402. (in Chinese)